

Constraining Interactions in Cosmology's Dark Sector

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We consider the cosmological constraints on theories in which there exists a nontrivial coupling between the dark matter sector and the sector responsible for the acceleration of the universe, in light of the most recent supernovae, large scale structure and cosmic microwave background data. For a variety of models, we show that the strength of the coupling of dark matter to a quintessence field is constrained to be less than 7% of the coupling to gravity. We also show that long range interactions between fermionic dark matter particles mediated by a light scalar with a Yukawa coupling are constrained to be less than 5% of the strength of gravity at a distance scale of 10 Mpc. We show that all of the models we consider are quantum mechanically weakly coupled, and argue that some other models in the literature are ruled out by quantum mechanical strong coupling.

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I. INTRODUCTION

Multiple, complementary cosmological observations all suggest that the Universe has recently embarked upon an epoch of accelerative expansion. These observations include the cosmic microwave background (CMB), for example Refs. [1, 2, 3, 4, 5, 6], large scale structure surveys, for example Refs. [7, 8, 9], including baryon acoustic oscillations [10, 11], and Type Ia supernovae [12, 13, 14, 15, 16]. There now appears to be irrefutable evidence that the Universe's expansion deviates from that predicted by Einstein's General Relativity and a Universe solely populated by baryonic matter and radiation.

Two new components, dark matter, that does not interact with light but does cluster under the force of gravity, and dark energy, that drives cosmic acceleration, have been invoked to resolve the disparities. In the minimal picture, dark matter does not feel any significant interactions, even with itself, apart from through gravity, and dark energy is a cosmological constant, not evolving and having no spatial fluctuations. Although this picture is wholly consistent with observations, the theoretical origin of both of these dark additions still remains a mystery, and the simple interpretation above has its own issues, such as the coincidence and fine tuning cosmological constant problems.

Recognizing that the physics of the dark sector is effectively unknown at present, and in light of the possible complexity of the dark sector arising out of high energy theory, theoretical models beyond the minimal picture have been considered. This includes a plethora of fundamental dark matter particle candidates, see for example [17] for a review, that might well be ex-

pected to have interactions beyond purely gravitational ones [18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. Such interactions can have astrophysical consequences, for example, the prospect of dark matter interactions, such as self-annihilation, that could give rise to the 511 keV emission [28]; the 'WMAP haze' [29, 30, 31]; implications for tidal streams in galactic systems [32, 33], as well as modifications to dark matter halo profile [34, 35, 36], dark matter halo mass function [37] or altered dark matter motion in cluster collisions, such as the Bullet Cluster [38].

One possibility that could mitigate the cosmological constant problems is that non-minimal interactions extend more broadly between dark sector particles, so that the properties of dark energy and dark matter are coupled in some way. Such a direct coupling can be employed to address the coincidence problem, by relating the onset of cosmic acceleration with the properties of a matter dominated universe [19, 23, 39, 40, 41, 42, 43, 44, 45, 46]. They can, however, also give rise to dynamical instabilities in the growth of structure [47, 48, 49, 50, 51, 52].

The paper proceeds as follows: in section II we describe two examples of dark sector interactions, coupled dark matter-dark energy models in II A and the Yukawa dark matter interaction in II B, that can have astrophysically observable consequences. In II C we summarize the theoretical and observational constraints on dark sector interactions, a subset of which we focus in on detail in the paper. We present the constraints from the latest cosmological observations on coupled dark matter-dark energy models in section III and the Yukawa dark matter interaction in IV. In section V we discuss the restrictions placed on models in the strong coupling regime. Finally, we pull together our findings and discuss their implications in section VI.

II. INTERACTING DARK MATTER

In this paper we consider scenarios in which a purely dark sector interaction exists, resulting from a non-

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minimal coupling of dark matter to a scalar field. Such couplings give rise to additional forces on dark matter particles in addition to gravity. In this section we describe two examples of models that exhibit this behavior. In the following sections we will discuss the observational constraints on these models.

A. Coupling dark matter to dark energy

Consider the general action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right] + \sum_j S_j \left[e^{2\alpha_j(\phi)} g_{\mu\nu}, \Psi_j \right], \quad (2.1)$$

where $g_{\mu\nu}$ is the metric, $M_p = (8\pi G)^{-1/2}$ is the reduced Planck mass, and we use natural units with $\hbar = c = 1$. Here ϕ is a scalar field which acts as dark energy, Ψ_j are the matter fields in the j th sector described by the action S_j , and $\alpha_j(\phi)$ describes the coupling of the scalar field to the j th sector. This general action (2.1) describes a wide range of models, including the Einstein frame version of $f(R)$ modified gravity [53, 54, 55, 56, 57]. A special case is when the couplings are identical in all the different sectors, $\alpha_j(\phi) = \alpha(\phi)$ for all j , in which case the theory satisfies the weak equivalence principle.

Although violations of the equivalence principle are strongly observationally constrained for normal matter, the constraints on dark matter are much weaker, as emphasized by Damour, Gibbons and Gundlach [21]. Therefore it is interesting to consider models with two sectors, dark matter with coupling function $\alpha_c(\phi)$, and normal (baryonic) matter with coupling function $\alpha_b(\phi)$. Such models will automatically satisfy observational constraints on the weak equivalence principle that involve only baryonic matter. They must also satisfy the additional constraint from Solar System observations that

$$M_p \alpha'_b(\phi_0) \lesssim 10^{-2}, \quad (2.2)$$

where ϕ_0 is the present day cosmological background value of ϕ ¹. Below we will specialize to models with $\alpha_b \equiv 0$, in which the scalar field is coupled only to the dark matter, which automatically satisfy the solar system constraint (2.2).

We note that theories of the form (2.1), in which different sectors couple in different ways to the scalar field ϕ , arise very naturally from higher dimensional models with branes. An example is provided by the Randall Sundrum I (RSI) model [63], with two parallel branes in a five dimensional anti-deSitter space, one with positive tension

and one with negative tension. The low energy four dimensional description of this model is of the form (2.1) with no potential [64, 65], with two sectors corresponding to matter on the two different branes, which we will denote $+$ and $-$. In this case the scalar field ϕ is a radion field that encodes the distance between the two branes in the fifth dimension. The two coupling functions are

$$\alpha_+(\phi) = \ln \cosh(\phi/\sqrt{6}M_p), \quad (2.3a)$$

$$\alpha_-(\phi) = \ln \sinh(\phi/\sqrt{6}M_p). \quad (2.3b)$$

The conventional interpretation of this RSI model is that visible matter lives on the negative tension brane, and that the positive tension (“Planck”) brane contains a hidden sector. This interpretation requires that the radion be stabilized, otherwise the Solar System constraint (2.2) is violated for all values of the present day cosmological value ϕ_0 of the scalar field. An alternative interpretation (which unlike the conventional one does not solve the hierarchy problem) is that visible matter is on the positive tension brane and that dark matter is on the negative tension brane, i.e., we make the identifications $+$ = b and $-$ = c , and the radion is not stabilized. In this model, normal matter is minimally coupled in the limit of small ϕ_0 (corresponding to distant branes), so that the constraint (2.2) can be satisfied in that regime.

In the remainder of this paper we assume zero baryonic-scalar coupling, $\alpha_b = 0$, and we will denote the dark matter coupling function $\alpha_c(\phi)$ simply as $\alpha(\phi)$. The model will then be specified completely by a choice of coupling function $\alpha(\phi)$ and potential $V(\phi)$.

B. Yukawa interaction between dark matter particles

Rather than coupling dark matter to dark energy, we can also modify the coupling of dark matter particles with themselves. One class of models of this type involve an interaction between fermionic dark matter, ψ , and an ultra-light pseudo scalar boson, ϕ , that interacts with the dark matter through a Yukawa coupling with strength g , described by the Lagrangian [34],

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\nabla^\mu\psi - m_\psi\bar{\psi}\psi - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + g\phi\bar{\psi}\psi. \quad (2.4)$$

For $g \neq 0$, on scales smaller than $r_s = m_\phi^{-1}$, the Yukawa interaction acts like a long-range ‘fifth’ force in addition to gravity. The effective potential felt between two dark matter particles is

$$V(r) = -\frac{Gm_\psi^2}{r} \left[1 + \alpha_{\text{Yuk}} \exp\left(-\frac{r}{r_s}\right) \right], \quad (2.5)$$

with

$$\alpha_{\text{Yuk}} \equiv 2g^2 \frac{M_p^2}{m_\psi^2}. \quad (2.6)$$

¹ This assumes that the solar perturbation to ϕ is in the linear regime, which is not true, for example, in chameleon models [58, 59, 60, 61, 62].

In our investigations of this model in Sec. IV we will neglect the cosmological effects of the scalar field ϕ , and assume that dark energy is a cosmological constant². The cosmological implications of Yukawa-like interactions of dark matter particles have previously been considered across a range of astrophysical scales, including dark matter halos [22, 66, 67], tidal tails [32, 33], cluster dynamics [38], and large scale structure surveys [68].

C. Theoretical and observational constraints

Models such as the ones described above face a range of theoretical and observational constraints arising from both particle physics and gravity. We will focus on a subclass of these in this paper, but it is worth mentioning the general web of desiderata and constraints. These include:

- *The existence of an ultraviolet (UV) completion.* Ideally one would like to find an embedding of the theory (2.1) in string theory. Such embeddings have been recently found for inflationary models, see, for example, the review [69]. However it is difficult to find UV completions for quintessence models; see, for example, the supergravity no-go theorem in Ref. [70].
- *Fine tuning and the taming of loop corrections.* Typically one would like a dark energy model to provide the unnaturally small value of the vacuum energy today. Having chosen such a small parameter value in one's Lagrangian, it is often necessary to fine tune the model to prevent renormalization of parameters through couplings to other fields. This is sometimes avoided in dark energy models by making the dark energy field a pseudo-Nambu-Goldstone boson, such as in the Yukawa scenario discussed in IIB. This is not necessarily the case for the action (2.1). In this paper we shall just assume that such tunings exist in (2.1), since avoiding them is not our focus.
- *The strong coupling problem.* If we treat the Lagrangian (2.1) as an effective field theory (as we should), valid up to some energy scale Λ , then there will exist irrelevant operators suppressed by powers of the cutoff. In certain regimes, these operators may become important, meaning that we are

no longer able to trust the effective theory. This will not arise in the theories we discuss here in a cosmological context. This strong coupling issue is discussed below in Sec. V.

- *Disagreement with the required background cosmology.* Obviously, a successful model must be able to reproduce the correct expansion history of the universe, preferably without excessive fine tuning of initial conditions. This can be a real problem for some models, for example some $f(R)$ modified gravity models [71]. In sections III and IV we investigate cosmological evolution in coupled models.
- *Problems with linear perturbations around the FRW solution.* Here the possibilities include disagreements with solar system tests of gravity [56], or incorrect predictions for the linear power spectrum of matter perturbations. In addition instabilities causing catastrophic collapse of over-densities can be present in some regimes for coupled theories [47, 48, 49, 50, 51, 52].
- *Problems in the nonlinear regime* There is the also possibility of interesting phenomena in the nonlinear regime. Some may be positive; for example the Chameleon effect [58, 59] can ameliorate problems with Solar System tests [72, 73]. Some other phenomena can be problematic, for example in some models the spatially averaged metric is not a solution of the field equations that one obtains by assuming homogeneity and isotropy (i.e. the “microscopic” and “macroscopic” field equations differ) [74, 75].

In this paper we will focus on the constraints obtained from the background cosmological evolution, linearized cosmological perturbations, and the strong coupling constraint.

III. COSMOLOGICAL CONSTRAINTS ON COUPLINGS BETWEEN DARK MATTER AND DARK ENERGY

In this section we consider the class of models (2.1) specialized to two sectors, the visible sector with zero coupling function, and the dark matter sector with coupling $\alpha(\phi)$ [19, 23, 40, 41, 42, 43, 45, 48, 50, 51]. The resulting equations of motion are

$$M_p^2 G_{ab} = T_{ab} + \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} (\nabla \phi)^2 - V(\phi) g_{ab} + e^{\alpha(\phi)} \rho_c u_a u_b, \quad (3.1a)$$

$$\nabla_a \nabla^a \phi - V'(\phi) = \alpha'(\phi) e^{\alpha(\phi)} \rho_c, \quad (3.1b)$$

$$\nabla_a (\rho_c u^a) = 0, \quad (3.1c)$$

$$u^b \nabla_b u^a = -\alpha'(\phi) (g^{ab} + u^a u^b) \nabla_b \phi. \quad (3.1d)$$

Here G_{ab} is the Einstein tensor, T_{ab} is the stress-energy tensor of visible matter and u^a is the four velocity of

² We note that the action (2.4) is actually a specific case of our general action (2.1), with $V(\phi) = m_\phi^2 \phi^2/2$ and $\alpha(\phi) = \ln[1 - g\phi/m_\psi]/3$, and specialized to the regime where the fermions are non-relativistic so that we can neglect the modifications to the fermion kinetic term in the action. However, our interpretation of this model is different from our interpretation of the models (3.9) and (3.16), since the ϕ field is not the dark energy and we are neglecting its cosmological evolution.

the dark matter. The quantity ρ_c is proportional to the number density of dark matter particles with respect to the metric g_{ab} ; it scales $\propto a^{-3}$ like uncoupled dark matter in the background cosmological solution. The observed energy density of dark matter is $e^\alpha \rho_c$.

A. Evolution of background cosmology

Writing the flat FRW metric as

$$ds^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2),$$

with scale factor $a(\tau)$ and conformal time τ , the Friedmann equation is

$$3M_p^2 \mathcal{H}^2 = \frac{1}{2} \dot{\phi}^2 + a^2 V(\phi) + a^2 e^{\alpha(\phi)} \rho_c + a^2 \rho_b + a^2 \rho_r, \quad (3.2)$$

where dots represent derivatives with respect to τ and $\mathcal{H} \equiv \dot{a}/a$. Here ρ_b and ρ_r are the densities of baryons and radiation. The remaining equations for the system are

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 V'(\phi) = -a^2 \alpha'(\phi) e^{\alpha(\phi)} \rho_c, \quad (3.3a)$$

$$\dot{\rho}_c + 3\mathcal{H}\rho_c = 0, \quad (3.3b)$$

$$\dot{\rho}_b + 3\mathcal{H}\rho_b = 0, \quad (3.3c)$$

$$\dot{\rho}_r + 4\mathcal{H}\rho_r = 0, \quad (3.3d)$$

where primes denote derivatives with respect to ϕ .

1. Dynamical attractors in general coupled models

Scalar field quintessence models of dark energy have been shown to have expansion histories that exhibit scaling attractor solutions which reduce sensitivity to initial conditions for the scalar field [39, 76, 77, 78, 79]. The same has been found to be true of coupled quintessence scenarios [19, 40, 41, 42], $f(R)$ gravity [71] and scalar-tensor gravity [80].

We specialize to the matter dominated era and neglect the baryons and radiation. To describe the attractor behavior in coupled models described by Eqs. (3.2)-(3.3), we use the dimensionless variables defined by Copeland et. al. [39, 46]:

$$x \equiv \frac{\dot{\phi}}{\sqrt{6}\mathcal{H}M_p}, \quad y \equiv \frac{a\sqrt{V}}{\sqrt{3}\mathcal{H}M_p}, \quad \lambda \equiv -\frac{M_p V'}{V}, \quad \Gamma \equiv \frac{VV''}{V'^2}, \quad (3.4)$$

along with the dimensionless coupling variable,

$$C(\phi) \equiv -\frac{M_p \alpha'}{\beta} \quad (3.5)$$

with $\beta \equiv \sqrt{2/3}$. Rewriting the evolution equations (3.2) – (3.3a) in terms of these variables and in terms of the dependent variable $N = \ln(a)$, with baryons and radiation

dropped, yields

$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2} \lambda y^2 + \frac{3}{2} x(1 + x^2 - y^2) + C(1 - x^2 - y^2), \quad (3.6a)$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2} \lambda xy + \frac{3}{2} y(1 + x^2 - y^2), \quad (3.6b)$$

$$\frac{d\lambda}{dN} = -\sqrt{6} \lambda^2 (\Gamma - 1)x. \quad (3.6c)$$

In these equations, Γ and C are understood to be the functions of λ obtained by eliminating ϕ in Eqs. (3.4) and (3.5). The fixed points of this system are the solutions of the equations $dx/dN = dy/dN = d\lambda/dN = 0$.

After Eqs. (3.6) have been solved to obtain the functions $x(N)$, $y(N)$ and $\lambda(N)$, the Hubble parameter $\mathcal{H}(N)$ can be found from Eqs. (3.4), and the dark matter density ρ_c can be obtained from the Friedmann equation,

$$x^2 + y^2 + \frac{a^2 e^\alpha \rho_c}{3M_p^2 \mathcal{H}^2} = 1. \quad (3.7)$$

Note that the effective total equation of state parameter w_{eff} , defined by $d \ln a / d \ln \tau = 2/(1 + 3w_{\text{eff}})$, is simply given by

$$w_{\text{eff}} = x^2 - y^2, \quad (3.8)$$

from Eqs. (3.4) – (3.7).

We consider the dynamical behavior for two specific models, with an exponential and power law potential, in the presence of an exponential coupling between the scalar field and cold dark matter.

2. Model 1: An exponential potential

We will consider a model with an exponential potential and linear coupling given by

$$V(\phi) = V_0 \exp\left(-\frac{\lambda\phi}{M_p}\right), \quad (3.9a)$$

$$\alpha(\phi) = -\frac{C\beta\phi}{M_p}. \quad (3.9b)$$

Here λ and C are dimensionless constants of order unity, and V_0 is a constant of order $M_p^2 H_0^2$. For this model the functions $\Gamma(\phi)$, $\lambda(\phi)$ and $C(\phi)$ defined by Eqs. (3.4) and (3.5) are constants:

$$\Gamma(\phi) = 1, \quad \lambda(\phi) = \lambda, \quad C(\phi) = C. \quad (3.10)$$

There exist three fixed points (these are a , b_m , c_m from Amendola's analysis of this specific model [41]):

$$(x, y) = \left[\frac{2C}{3}, 0 \right], \quad (3.11a)$$

$$(x, y) = \left[\frac{\lambda}{\sqrt{6}}, \left(1 - \frac{\lambda^2}{6} \right)^{1/2} \right], \quad (3.11b)$$

$$(x, y) = \left[\left(\frac{3}{2} \right)^{1/2} \frac{1}{\lambda - \beta C}, \left(\frac{3}{2} \right)^{1/2} \frac{1}{\lambda - \beta C} \times (1 + \beta^4 C^2 - \beta^3 C \lambda)^{1/2} \right]. \quad (3.11c)$$

It is important to note that, depending on the values of the parameters of the model, some of these fixed points may not exist, i.e., they may be complex rather than real. In addition, when they do exist, they may or may not be stable attractors during the matter and dark energy era.

The first of these (3.11a) is an attractor approached as the matter dominated era is entered. The potential is subdominant and the scalar field kinates, leading to an effective equation of state parameter

$$w_{\text{eff}} = \frac{4C^2}{9}. \quad (3.12)$$

This evolution is often described as a ' ϕ CDM' era, and differs from the usual CDM dominated era with $w_{\text{eff}} = 0$. Its existence and properties have led to significant issues when fitting some $f(R)$ theories, for which $C = 1/2$, to observations [71].

The second fixed point, (3.11b), is a stable attractor for

$$\lambda(\lambda - \beta C) < 3 \quad (3.13)$$

with effective equation of state parameter

$$w_{\text{eff}} = -1 + \frac{\lambda^2}{3}. \quad (3.14)$$

This attractor gives rise to acceleration if $\lambda^2 < 2$. This fixed point arises entirely from the nature of the scalar potential, and is independent of the coupling C ; in particular it arises in the minimally coupled case $C = 0$.

The final fixed point, (3.11c), with

$$w_{\text{eff}} = \frac{\beta C}{\lambda - \beta C}, \quad (3.15)$$

exists if the second fixed point is unstable. We will find, however, that condition (3.13) is satisfied in the viable models we analyze below, so that this final fixed point does not arise.

3. Model 2: A power law potential

We also consider the inverse power law potential model

$$V(\phi) = V_0 \exp\left(\frac{M_p}{\phi}\right)^n, \quad (3.16a)$$

$$\alpha(\phi) = -\frac{C\beta\phi}{M_p}, \quad (3.16b)$$

where n is a constant for which

$$\Gamma(\phi) = \frac{n+1}{n}, \quad \lambda(\phi) = -n \left(\frac{M_p}{\phi} \right), \quad C(\phi) = C. \quad (3.17)$$

There are two stable attractors which arise in the matter and accelerated eras, respectively,

$$(x, y) = \left[\frac{2C}{3}, 0 \right], \quad (3.18)$$

$$(x, y) = [0, 1], \quad (3.19)$$

for which, in both cases, $\lambda \rightarrow 0$. Eq. (3.18) gives a matter dominated era attractor equivalent to (3.12), while (3.19) is an accelerative attractor with $w_{\text{eff}} = -1$, independent of C and n .

4. Numerical evolution of attractors

In Figure 1 we show the background expansion history for examples of the exponential and power law potentials and the coupling discussed here.

Typically in these models, the radiation era evolution is the same as in Λ CDM, with scalar field attractors with $\Omega_\phi = 0$ or $w_\phi = 1/3$. In certain cases, e.g. exponential models with $\lambda \geq 2$, however, the radiation era can be replaced by a kinetic scalar field dominated era for models with H_0 consistent with HST. However these models do not confront data well.

A difference between Λ CDM and the coupled scenarios can arise in the matter dominated era as described above. In this regime the attractor evolution alters the matter dominated expansion history via equation (3.12). The angular diameter distance of the CMB and the growth functions for large scale matter perturbations ($k < k_{\text{eq}}$) entering the horizon after matter radiation equality, relative to the smaller scale ($k > k_{\text{eq}}$) perturbations, is altered in comparison to Λ CDM.

At late times, the coupled models tend towards accelerative attractors which are independent of the coupling C , given by (3.11b) for the exponential and (3.19) for the power law potentials, respectively. Note, however, that the evolution will not necessarily have reached the attractor today, and the coupling can therefore play a role in determining w_{eff} by altering the time at which the shift from the ϕ CDM to the accelerative attractor occurs.

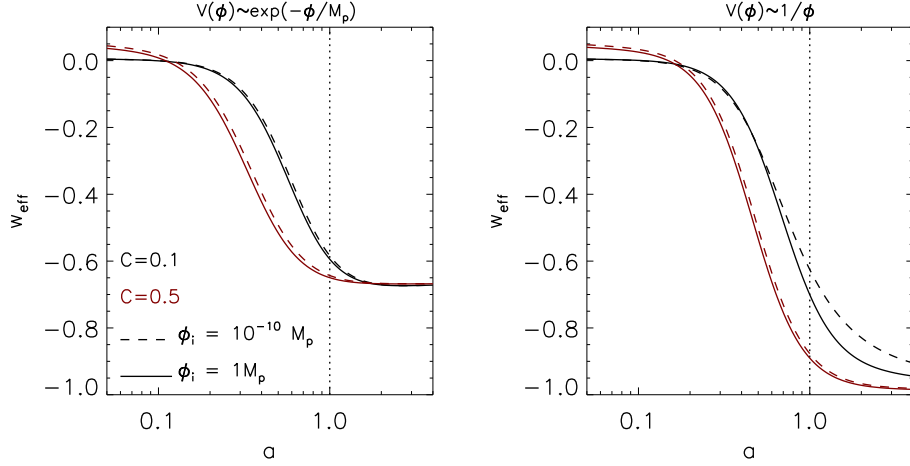


FIG. 1: Examples of evolution of the effective equation of state, w_{eff} , in coupled scalar field dark matter models with an exponential potential $V(\phi) \propto \exp(-\phi/M_p)$ (left panel) and a power law potential $V(\phi) \propto 1/\phi$ (right panel). Cosmological parameters are fixed to $H_0 = 70$, $\Omega_c = 0.25$, $\Omega_b = 0.05$, and $C = 0.1$ (black) and $C = 0.5$ (red). Both models follow the coupling dependent attractor in the matter dominated era and asymptote to coupling independent attractors at late times. The timing of the transition between these two attractors is sensitive to both the potential and coupling parameters. For the exponential potential the dynamical attractor leads to a negligible dependence on initial conditions, shown here through comparing evolution with two different initial values of $\phi_i \equiv \phi(a = 10^{-8})$, $\phi_i = 1M_p$ (full) and $10^{-10}M_p$ (dashed). For the power law potential, however, a sensitivity to initial conditions can exist in the transition era. This is accounted for in the analysis by marginalizing over initial conditions.

As shown in Fig. 1, for the exponential potential the attractor behavior quickly takes over, and the initial conditions have no effect on the dynamical evolution. In the case of the power law potential, however, we find there can still remain some sensitivity to the initial value of the scalar field during the transition between matter dominated and accelerative attractors. As discussed in section III C, we account for this in the analysis by marginalizing over the initial value of ϕ .

B. Evolution of linearized cosmological perturbations

As well as background evolution, we are also interested in the predicted evolution of density perturbations. We write the inhomogeneous density and scalar field as

$$\rho_c(x, \tau) = \rho_c(\tau)(1 + \delta_c(x, \tau)), \quad (3.20a)$$

$$\phi(x, \tau) = \phi(\tau) + \varphi(x, \tau). \quad (3.20b)$$

We use the notation of Ref. [81] to describe the perturbed metric in synchronous gauge in terms of two functions $\eta(\tau)$ and $h(\tau)$. The four independent components of the Einstein equation are then

$$2k^2\eta - \mathcal{H}\dot{h} = -a^2e^\alpha\rho_c(\delta + \alpha'\varphi) - a^2V'\varphi - \dot{\phi}\dot{\varphi}, \quad (3.21a)$$

$$2k^2\dot{\eta} = a^2e^\alpha\rho_c\theta_c + k^2\dot{\phi}\varphi, \quad (3.21b)$$

$$\ddot{h} + 2\mathcal{H}\dot{h} - 2k^2\eta = -3\dot{\phi}\dot{\varphi} + 3a^2V'\varphi, \quad (3.21c)$$

and

$$6\ddot{\eta} + \ddot{h} + 2\mathcal{H}(\dot{h} + 6\dot{\eta}) - 2k^2\eta = 0. \quad (3.22)$$

Here k is the comoving wavevector and $\theta_c = ik_j v_c^j$ is the gradient of the CDM peculiar velocity, v_c . Also we have specialized to units with $M_p = 1$. We include just the effects of CDM and the scalar field, and neglect baryons and radiation, for simplicity. The perturbed fluid equations are

$$\dot{\delta}_c + \frac{1}{2}\dot{h} + \theta_c = 0, \quad (3.23)$$

$$\frac{d}{d\tau}(ae^\alpha\theta_c) = ak^2\alpha'e^\alpha\varphi, \quad (3.24)$$

$$\begin{aligned} \ddot{\varphi} + 2\mathcal{H}\dot{\varphi} + [k^2 + a^2V'' + a^2e^\alpha\rho_c(\alpha'' + (\alpha')^2)]\varphi \\ = -\frac{1}{2}\dot{h}\dot{\phi} - a^2\alpha'e^\alpha\delta_c\rho_c. \end{aligned} \quad (3.25)$$

There exists an extra gauge degree of freedom that preserves synchronous gauge, given by the coordinate transformations

$$\tau \rightarrow \tau + \frac{c_0}{a}\mathcal{R}[e^{i\mathbf{k}\cdot\mathbf{x}}], \quad (3.26)$$

$$x^j \rightarrow x^j + kc_0\mathcal{R}[\hat{k}_je^{i\mathbf{k}\cdot\mathbf{x}}] \int \frac{d\tau}{a}, \quad (3.27)$$

where c_0 is a constant and $\hat{k}_k = k_j/k$. Under this transformation the metric and matter perturbations transform

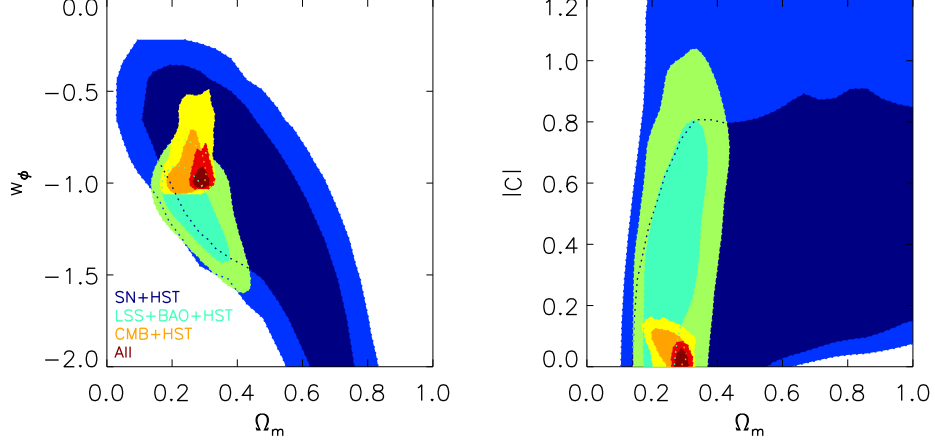


FIG. 2: Joint 68% (dark shaded) and 95% (light shaded) constraints in the exponential potential model for the the fractional matter density, Ω_m and the effective scalar equation of state, w_ϕ (left panel), and the coupling, C (right panel). The complementary constraints arising separately from the WMAP CMB spectra, SDSS matter power spectrum plus SDSS and 2dFGRS baryon acoustic oscillation data sets, and the ‘union’ Type 1a supernovae data set, and a HST prior on H_0 are shown.

as

$$\eta \rightarrow \eta + \mathcal{H} \frac{c_0}{a}, \quad (3.28)$$

$$h \rightarrow h - 6\mathcal{H} \frac{c_0}{a} + 2k^2 c_0 \int \frac{d\tau}{a}, \quad (3.29)$$

$$\theta_c \rightarrow \theta_c - c_0 \frac{k^2}{a}, \quad (3.30)$$

$$\varphi \rightarrow \varphi - c_0 \frac{\dot{\phi}}{a}, \quad (3.31)$$

$$\delta_c \rightarrow \delta_c + 3\mathcal{H} \frac{c_0}{a}. \quad (3.32)$$

We can define two new variables

$$\delta_{*c} = \delta_c + 3\mathcal{H} \frac{\theta_c}{k^2} \quad (3.33a)$$

$$\varphi_* = \varphi - \frac{\dot{\phi}}{k^2} \theta_c \quad (3.33b)$$

that are invariant under the residual gauge transformations.

In the minimally coupled case $\alpha(\phi) = 0$, one typically fixes the residual gauge freedom by choosing the CDM rest frame in which $\theta_c = 0$ and $\delta_c = \delta_{*c}$ [81]. This is consistent since, once fixed to zero, θ_c remains zero at all times, by Eq. (3.24). In the presence of an evolving non-minimal coupling, however, if the CDM velocity divergence is initially zero it will evolve to be non-zero. Therefore it is not possible to fix the residual gauge freedom in this way. In our computations below we will evolve the perturbation equations in an arbitrary synchronous gauge, and then use Eqs. (3.33) to pick out gauge invariant combinations of the perturbation variables.

C. Comparison with data

We have modified the CAMB code [82] to evolve background equations and first order density perturbations for a flat universe containing baryons, CDM, radiation, massless neutrinos and a scalar field coupled non-minimally to CDM and use CosmoMC [82] to perform a Monte Carlo Markov chain of the model parameter space in comparison to current cosmological data. We explore the exponential model (3.9) and the power law model (3.16) allowing the exponent λ and index n to vary in each case. We set the initial conditions for the scalar field well into the radiation era, $a = 10^{-8}$, allowing the initial value of the scalar field to vary, then evolve forward to the times at which CAMB typically begins its integration.

We constrain the models using a combination of cosmological datasets, including the measurements of the CMB temperature and polarization power spectrum from the WMAP 5-year data release [6, 83], the ‘union’ set of supernovae compiled by the Supernovae Legacy Survey (SNLS) [16], and we impose a Gaussian prior on the Hubble constant today, $H_0 = 72 \pm 8$, using the Hubble Space Telescope (HST) measurements [84]. We use the matter power spectrum of Luminous Red Galaxies (LRG) as measured by the Sloan Digital Sky Survey (SDSS) survey [8, 9], for which we include the shift parameter, a_{scl} , to adjust the matter power spectrum as discussed in [8],

$$a_{scl} = \frac{d_V(z=0.35)^{(model)}}{d_V(z=0.35)^{(fiducial)}} \quad (3.34)$$

$$d_V = \left[\frac{(1+z)^2 d_A(z)^2 c z}{H(z)} \right]^{1/3} \quad (3.35)$$

where $d_A(z)$ is the physical angular diameter distance at a redshift z and the fiducial model is a standard Λ CDM

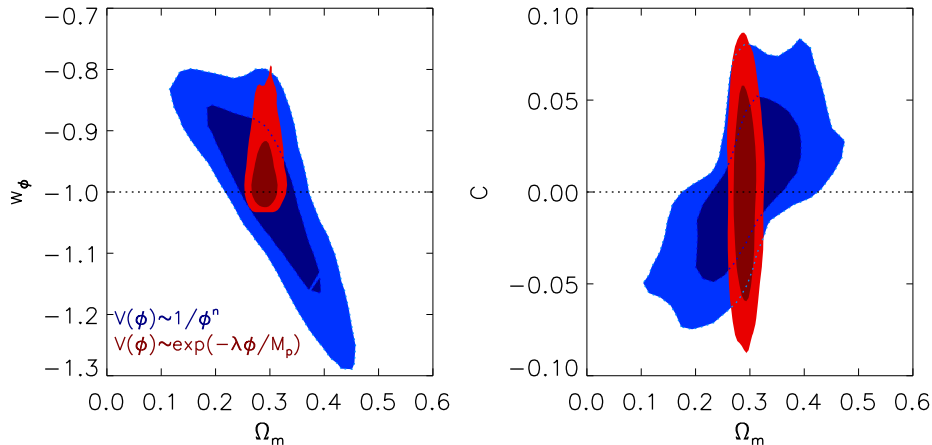


FIG. 3: Joint 68% (dark shaded) and 95% (light shaded) constraints using combined WMAP CMB, SDSS matter power spectrum, SDSS and 2dFGRS baryon acoustic oscillation, ‘union’ type Ia supernovae datasets and HST H_0 prior for the power law potential (blue) and exponential potential (red) for the fractional matter density, Ω_m and the effective scalar equation of state, w_ϕ , (left panel), and the coupling, C (right panel).

model with $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$ and with the same Hubble constant as the theory model. We also use constraints on the expansion history from the Baryon Acoustic Oscillation data of the 2dFGRS and SDSS surveys [11], based on measurements of the ratio of the sound horizon at last scattering, $r_s(z_*)$, to the distance measure $d_V(z)$ at $z = 0.2$ and $z = 0.35$. Since the dynamical attractor solutions, in the presence of a non-minimal coupling, can alter the background evolution in the matter dominated era, one finds that the redshift of last scattering, z_* , can no longer be accurately estimated using the fitting formula of Hu and Sugiyama [85]. Instead we calculate the redshift of maximum visibility and use this as the appropriate measure for the redshift of last scattering.

In Figure 2 we show the complementary 2D marginalized constraints for the exponential potential model in light of the various cosmological datasets. The CMB data (along with the HST prior on H_0) provide the best individual constraint on the coupling strength with 1D marginalized constraints $|C| \leq 0.13$ at the 95% confidence level (c.l.). The Type Ia supernovae alone provide only weak constraints on both the coupling and on the total matter density in a non-minimally coupled model. This is because the coupling allows a late time, cosmologically consistent expansion with $w_{\text{eff}} \approx \Omega_\phi w_\phi \sim -0.7$ to be generated by a strongly phantom-like model, with $w_\phi \ll -1$ and $\Omega_\phi \approx 0$, where

$$w_\phi \equiv \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi) + (e^{\alpha\phi} - 1)\rho_c} \quad (3.36)$$

$$\Omega_\phi \equiv \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi) + (e^{\alpha\phi} - 1)\rho_c}{\frac{1}{2}\dot{\phi}^2 + V(\phi) + e^{\alpha\phi}\rho_c + \rho_b + \rho_\gamma}. \quad (3.37)$$

These models are not consistent with CMB and LSS observations, however.

In Figure 3 we show 2D marginalized constraints for the exponential and power law potential models from all the cosmological datasets combined. The 1D marginalized constraint on the coupling in the exponential potential case is $|C| < 0.037(0.067)$ at the 68% (95%) confidence level. This represents a significant tightening of constraints over previous analyses, for example [86] found $|C| < 0.1$ at the 98.6% level using CMB data from the Boomerang satellite. The potential exponent, λ , is constrained to be $|\lambda| < 0.95$ at the 95% c.l..

In the power law potential case the 1D marginalized constraint on the coupling is comparable, with $-0.026(-0.055) \leq C \leq 0.034(0.066)$ at the 68% (95%) c.l.. Again, the constraints on the coupling strength have improved with the increased precision and complementary variety of the cosmological data, e.g. a previous analysis with first year WMAP data alone found $C \leq 0.085(0.159)$ at the 68% (95%) c.l. [87]. Within the range investigated, $-6 \leq n \leq 6$, the power law exponent, n , is not constrained by the data.

In both the exponential and power law potential cases the constraints are wholly consistent with a minimally coupled Λ CDM model ($\lambda = 0$ or $n = 0$, and $C = 0$) at the 1σ level.

IV. COSMOLOGICAL CONSTRAINTS ON A YUKAWA-TYPE DARK MATTER INTERACTION

The astrophysical implications of Yukawa-like interactions have been considered across a range of scales: in the context of dark matter halos [22, 66, 67]; tidal tails [32, 33]; cluster dynamics [38]; and large scale structure surveys [68]. In our analysis we consider large scale cosmological constraints on a Yukawa coupling described in

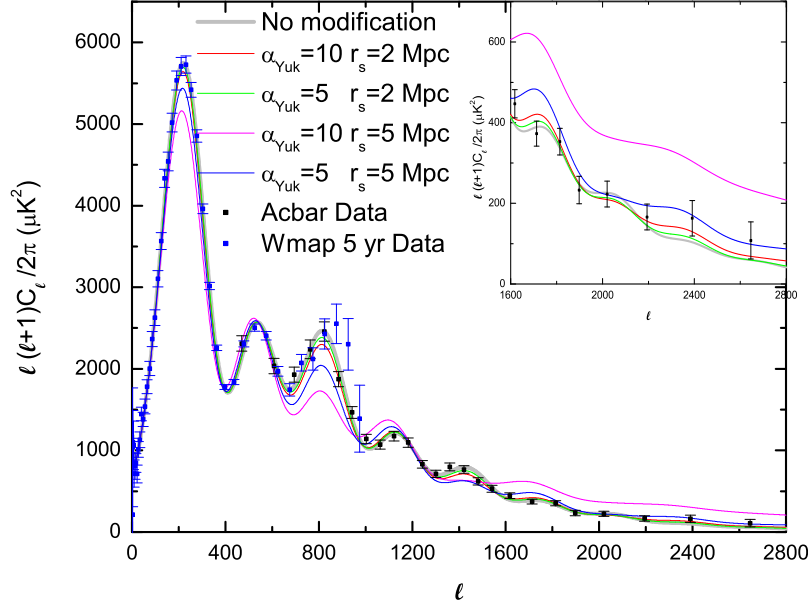


FIG. 4: CMB temperature power spectrum comparing a fiducial minimally coupled Λ CDM model (grey) with four models with the Yukawa interaction with $\alpha_{\text{Yuk}} = 10, r_s = 2 \text{ Mpc}$ (red), $\alpha_{\text{Yuk}} = 5, r_s = 2 \text{ Mpc}$ (green), $\alpha_{\text{Yuk}} = 10, r_s = 5 \text{ Mpc}$ (magenta), $\alpha_{\text{Yuk}} = 5, r_s = 5 \text{ Mpc}$ (blue). Data from WMAP 5 year (blue points) and ACBAR (black points) experiments are also shown. The inset plot shows a blow up of the small scale anisotropies measured by ACBAR.

section II B. We modify the publicly available CAMB code [88] to include this modified force between dark matter particles. This alters the growth of matter perturbations. For example, the dark matter density fluctuations evolve according to

$$\ddot{\delta}_c + \mathcal{H}\dot{\delta}_c - 4\pi G a^2 \left[\frac{G_c(k)}{G} \rho_c \delta_c + \rho_b \delta_b + 2\rho_\gamma \delta_\gamma \right] = 0. \quad (4.1)$$

Here $G_c(k)$ is the effective gravitational constant governing the interaction between dark matter particles, given from Eq. (2.5) by

$$G_c(k) = G \left[1 + \frac{\alpha_{\text{Yuk}}}{1 + (kr_s)^{-2}} \right]. \quad (4.2)$$

We use the CosmoMC code [82] to obtain cosmological constraints on the ratio G_c/G from the 5 year WMAP CMB temperature and polarization data [6, 83], small scale CMB temperature data from ACBAR [4, 5] and the SDSS LRG matter power spectrum [8]. We include CMB lensing, and marginalize over the amplitude of the secondary Sunayev-Zel'dovich anisotropies.

In Figure 4 we show the effect of the Yukawa coupling on the CMB temperature anisotropies. With the addition of small scale anisotropy measurements from ACBAR, constraints on the interaction are able to be made.

In Figure 5 we show the constraints on G_c/G at two scales, 1 Mpc and 10 Mpc with $G_c/G(1 \text{ Mpc}) \leq 2.7$ and $G_c/G(10 \text{ Mpc}) \leq 1.05$ at the 68% confidence limit. The improvement in the fit to the data obtained by introducing the Yukawa interaction is not statistically significant however, the best fit effective $\chi^2 = -2 \ln \mathcal{L} = 1354.0$ in comparison to 1354.1 for a Λ CDM model.

Yukawa interactions on the levels allowed by large scale constraints could well have interesting implications for gravitational dynamics on cluster, galactic and sub-galactic scales [32, 33, 34, 35]. Frieman and Gradwohl [34] argue that the intracluster gas distribution could constrain $-0.5 \lesssim \alpha_{\text{Yuk}} \lesssim 1.3$ for r_s of a few hundred kpc, which would translate to $-0.5 \lesssim G_c/G(1 \text{ Mpc}) \lesssim 2.2$, comparable with our constraints from large scale data. Kesden and Kamionkowski [32, 33] demonstrate that couplings of strength $G_c/G \gtrsim 1.04$ on $\lesssim 100 \text{ kpc}$ scales could well have observable implications for baryonic and dark matter distributions in tidal disruptions of dwarf galaxies, although a comparison with data is yet to be performed. We leave a detailed analysis of the joint constraints on Yukawa interactions from combined astrophysical and cosmological scales to future work.

We note that the observational constraints on the Yukawa coupling α_{Yuk} also yield constraints on the more general class of models (2.1) discussed in Sec. III, pa-

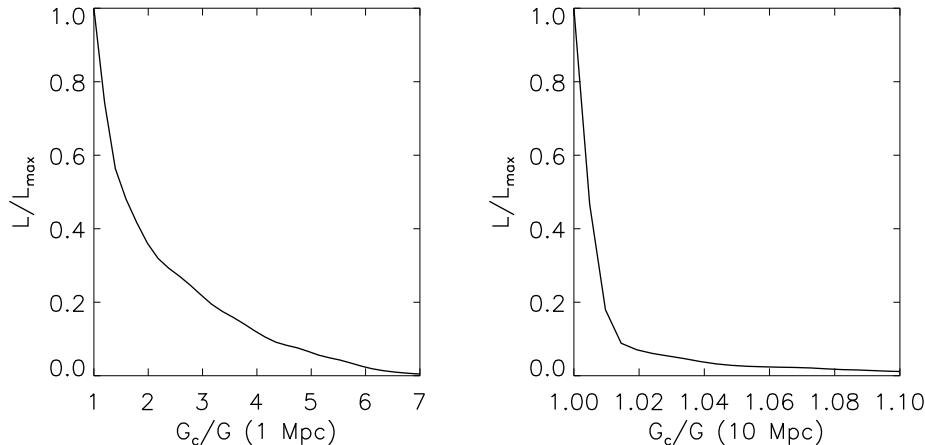


FIG. 5: 1D likelihood constraints on G_c/G at 1Mpc (left panel) and 10Mpc (right panel) for the Yukawa dark matter interaction, in light of WMAP 5 year and ACBAR CMB anisotropy, and SDSS LRG matter power spectrum observations.

parameterized by a baryonic coupling function $\alpha_b(\phi)$ and a dark matter coupling function $\alpha_c(\phi)$. In these models the effective Newton's constant G_{ij} for coupling between sector i and sector j is given by $G_{ij} = G(1 + \gamma_i\gamma_j)$ with $\gamma_i = \sqrt{2}M_p\alpha'_i(\phi_0)$ [51]. Now dark matter is observed only through its gravitational interactions. Therefore the observations cannot distinguish between a situation with baryonic and dark matter densities ρ_b , ρ_c and Newton's constants G_{cc} , G_{cb} and G_{bb} , and a situation with densities ρ_b , $e^\nu\rho_c$ and coupling constants $e^{-2\nu}G_{cc}$, $e^{-\nu}G_{cb}$ and G_{bb} , where ν is an arbitrary constant³. If we define the parameter

$$\begin{aligned}\alpha &\equiv \frac{G_{cc}G_{bb}}{G_{cb}^2} - 1 \\ &= \frac{(1 + \gamma_c^2)(1 + \gamma_b^2)}{(1 + \gamma_c\gamma_b)^2} - 1,\end{aligned}\quad (4.3)$$

then we see that α is invariant under the above symmetry, and also α reduces to α_{Yuk} for the models discussed in this section for which $\alpha_b = 0$, at short lengthscales $r \ll r_s$. It follows that the arguments of Ref. [34] give the constraint

$$-0.5 \lesssim \alpha \lesssim 1.3 \quad (4.4)$$

on the class of models (2.1). This constraint already significantly limits some models, for example together with the Solar System constraint (2.2) it rules out the version of the RSI model we discussed in Sec. II A above.

V. QUANTUM MECHANICAL STRONG COUPLING CONSTRAINT

General relativity is a weakly coupled effective quantum field theory at lengthscales large compared to the Planck length [89, 90]. However, many modifications of general relativity do not share this property. It can happen that at relatively low energies, loop corrections become large and one can no longer trust the classical theory. The theory becomes strongly coupled, like quantum chromodynamics at low energies. This occurs for theories of massive gravitons at energy scales above $(m_g^2 M_p)^{1/3}$, where m_g is the graviton mass and M_p is the Planck mass [91], and in the DGP model at lengthscales below ~ 1000 km [92]. It is also generic for theories which modify gravity in the infrared without introducing new degrees of freedom [93].

Many coupled cosmic acceleration models in the literature are invalid because of this consideration. There is a straightforward procedure for computing when a model of the form (2.1) is in the strong coupling regime: for a given classical solution, compute the action of fluctuations $\delta\phi$ about that classical solution, and then Taylor expand that action. The Taylor expansion of the potential gives nonrenormalizable terms of the form $\delta\phi^{n+4}/(\Lambda_n)^n$, where Λ_n is some energy scale and $n \geq 1$ is an integer. Then the theory is strongly coupled at energy scales above the lowest of the scales Λ_n , ie for $E \gtrsim \Lambda_{\text{sc}}$ where $\Lambda_{\text{sc}} = \min_n \Lambda_n$. If the corresponding lengthscale $r \sim 1/\Lambda_{\text{sc}}$ is in the range probed by observations, then the predictions of the classical theory cannot be used to compare with observations. We will see that some models are ruled out by this consideration.

For a generic scalar field theory which acts as a model for cosmic acceleration, the potential can be written as

$$V(\phi) \sim H_0^2 M_p^2 \bar{V}(\phi/M_p), \quad (5.1)$$

³ Note that G_{cc} is denoted G_c in the rest of this paper.

where the function \bar{V} is a dimensionless function of a dimensionless argument. For a generic model we expect the derivatives of \bar{V} to be of order unity, so the n th term in the Taylor expansion scales as $H_0^2 M_p^2 (\phi/M_p)^n$. The corresponding strong coupling scale Λ is $\Lambda \sim M_p (M_p/H_0)^{2/(4-n)}$ which is larger than M_p for $n \geq 4$. Thus for generic quintessence models there is no strong coupling issue just from the potential.

We next discuss the effects of matter coupling. Consider a generic theory of the form (2.1) with coupling function $\alpha(\phi)$ and potential $V(\phi)$, for which the potential contains a nonrenormalizable term

$$(\delta\phi)^{4+n}/\Lambda^n \quad (5.2)$$

with $n \geq 1$. We first show that, for a localized source of mass $\sim M$ and size $\sim R$, such a term has a significant effect classically before it leads to strong coupling, as long as the mass M is sufficiently large. Thus, in the regime where one can treat perturbations from matter inhomogeneities linearly, the theory is never strongly coupled.

To see this, we denote by $\delta\phi$ the perturbation to the cosmological background solution $\phi(t)$ that is generated by the localized source. We take the ratio of the terms $(\delta\phi)^{4+n}/\Lambda^n$ and $(\nabla\delta\phi)^2$ in the action to get

$$\sim \frac{(\delta\phi)^{2+n} R^2}{\Lambda^n}. \quad (5.3)$$

We assume that in the absence of the term (5.2) in the action, the scalar field can be treated as a massless field with dimensionless coupling strength C to matter, obeying an equation of the form

$$\square\delta\phi = -\beta C\rho/M_p. \quad (5.4)$$

This gives the following order of magnitude estimate (dropping factors of order unity, including β) for the value of the scalar field perturbation near the surface at $r \sim R$,

$$\delta\phi \sim \frac{|C|M}{M_p R}. \quad (5.5)$$

The ratio (5.3) will therefore be small for $\Lambda \gg \Lambda_{\text{cl}}$, where the critical value of Λ for the perturbation to the potential to be important classically is

$$\Lambda_{\text{cl}} \sim \frac{1}{R} \left(\frac{|C|M}{M_p} \right)^{1+2/n}. \quad (5.6)$$

Next, the theory will be strongly coupled at energies $E \gtrsim \Lambda$, and for a source of size $\sim R$ the relevant quanta have energies $E \sim 1/R$, so strong coupling will occur for $\Lambda \gtrsim \Lambda_{\text{sc}} \equiv 1/R$. Comparing this with the estimate (5.6), we see that as long as

$$|C|M \gtrsim M_p, \quad (5.7)$$

the theory will never be strongly coupled in the regime where the perturbation to the potential can be neglected

classically. The condition (5.7) will be satisfied for all astrophysically relevant sources when C is of order unity.

This analysis applies to a large class of scalar field theories, including the models discussed in this paper. However it does not apply to theories where nonlinearities are important classically, such as chameleon field models and $f(R)$ versions of these. The strong coupling constraint on these models must be checked on a case by case basis. For example, for the $f(R)$ theory of Faulkner et al. [72], an order of magnitude estimate gives the strong coupling scale to be $\Lambda_{\text{sc}} \sim M(M^3 M_p/\rho)^{1/(n+1)}$ where ρ is the matter density and M, n are parameters of the model. Demanding that Λ_{sc} not be so low as to invalidate the predictions of the model rules out a large portion of the parameter space in this case.

VI. CONCLUSIONS

The cosmological observations of the past few decades have provided firm evidence for significant physics beyond the standard model of particle physics. It now seems clear that the successful formation of structure in the universe demands a new particulate component of the cosmic energy budget - dark matter - and that cosmic acceleration may require some kind of dark energy, or a significant infrared modification of general relativity.

While these phenomena have been revealed through their gravitational effects, their microphysical properties remain undetermined although, of course, there exist many complementary bounds on what those properties may eventually prove to be. A priori, there is no reason that dark matter and the physics responsible for cosmic acceleration should find themselves in entirely disconnected sectors of our underlying fundamental theory. Indeed, that small portion of the energy budget about which we know a great deal - visible matter - comprises a richly detailed spectrum with multiple interactions and a beautiful underlying symmetry structure - the standard model. It thus seems reasonable, in light of our current ignorance regarding the nature of cosmology's dark sector, to explore possible interactions between dark matter, cosmic acceleration and visible matter.

In this paper we have studied a broad class of coupled dark matter dark energy models, and have investigated the constraints on such models from the most recent supernovae data, from precise measurements of the large scale structure of the universe, and from cosmic microwave background experiments, including the recent WMAP 5-year data release. While it is notable how constraining each of these sources is individually, the combined constraints are surprisingly strict. Indeed, for the class of models studied here, we have demonstrated that the strength of the coupling of dark matter to a quintessence field is constrained to be less than 7% of the coupling to gravity at the 95% confidence level.

Furthermore, we have applied our techniques to models of possible astrophysical interest, in which long range

interactions between fermionic dark matter is mediated by a light scalar with a Yukawa coupling. We have shown that large scale cosmological measurements constrain such interactions to be less than 5% of gravity at a distance scale of 10 Mpc.

We have also shown that the models considered here are weakly coupled throughout the relevant parts of the parameter space, unlike some other models in the literature that couple dark matter and dark energy and are ruled out by the quantum mechanical strong coupling.

It is a testament to the many diverse sources of data in modern cosmology that the simple possibility of couplings between, say, dark matter and dark energy, can be constrained in so many different ways. The web of constraints that we have delineated in this paper sets strict

limits on allowed interactions in the dark sector, and may have important ramifications both for phenomenological models, and for fundamental theory.

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